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Research Article

Robust Moving Horizon \mathcal{H}_∞ Control of Discrete Time-Delayed Systems with Interval Time-Varying Delays

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In this study, design of a delay-dependent type moving horizon state-feedback control (MHHC) is considered for a class of linear discrete-time system subject to time-varying state delays, norm-bounded uncertainties, and disturbances with bounded energies. The closed-loop robust stability and robust performance problems are considered to overcome the instability and poor disturbance rejection performance due to the existence of parametric uncertainties and time-delay appeared in the system dynamics. Utilizing a discrete-time Lyapunov-Krasovskii functional, some delay-dependent linear matrix inequality (LMI) based conditions are provided. It is shown that if one can find a feasible solution set for these LMI conditions iteratively at each step of run-time, then we can construct a control law which guarantees the closed-loop asymptotic stability, maximum disturbance rejection performance, and closed-loop dissipativity in view of the actuator limitations. Two numerical examples with simulations on a nominal and uncertain discrete-time, time-delayed systems, are presented at the end, in order to demonstrate the efficiency of the proposed method.

1. Introduction

Time-delay systems have drawn a considerable amount of attention in the last few decades due to the fact that these systems represent the behaviour of processes more close to real world situations. The primary effects of delay in the behaviour of physical systems are on deterioration of performance and exhibition of an unstable response. There are many sources of delay in a system which can be exemplified such as long transmission lines or intensive communication channels, approximations in the identification or modelling of real systems, and finite rate or capability of computing power for control and communication purposes in remote systems. In the literature, a great deal of studies have comprehensively investigated the time delayed systems in terms of their negative effects on the stability [1–3] and performance [4–6] of feedback systems.

\mathcal{H}_∞ control mainly deals with the synthesis of feedback or feed-forward controller for dynamical systems to establish a stable behaviour or respond in a robust sense when the controlled system is subjected to an external disturbance effect.

Particularly, in the last 2 decades, \mathcal{H}_∞ control problem of time-delay systems received a considerable amount of interest, which might be constant or even time-varying. The main reason arises from the fact that the so-called bounded real lemma (BRL) signifying the \mathcal{H}_∞ performance of a given plant or system devoid of any delay effect proves to yield a necessary condition while its counterpart given for the case of an existence of time-delay can afford to achieve only a sufficient condition. This observation implies that the \mathcal{H}_∞ results concerning time-delay systems inherently involve potential conservativeness which lead the researchers in the time-delay community to construct further investigation to achieve better \mathcal{H}_∞ performance. Therefore, for continuous time case, one can refer to [7–11] and the references therein for the \mathcal{H}_∞ control of time-delay systems (TDSs) having zero lower delay bound and [10, 12–16] for the \mathcal{H}_∞ control of TDSs with interval time-delays. For the discrete-time counterpart, the reader can refer to [17–19] and the references therein.

Moving horizon control (MHC), also called receding horizon control (RHC) or model predictive control (MPC)

in the literature, is a widely used method of control of industrial processes, especially having large number of variables and constraints, due to its superior capabilities in handling constraints on control and states, operating with less human intervention and reacting dynamically to system changes easily. The main approach in MPC is to solve a constraint optimization problem on-line at each time instant by utilizing the past and current feedback information and apply only the very recent control solution to the system. For a good survey and recent approaches on the concept, one can refer to [20–26] and the references therein. The general classification and areas of application of these systems have been comprehensively investigated in references [27, 28].

It is well known that uncertainties and time-delays cannot be avoided in real life, especially, in many processes such as chemical, biological, and network problems where the MPCs have been widely used. However, it is apparently seen from the literature that only very few results exist concerning the MPC of time-delay systems. Based on the L - K functional approach and the use of some relaxation matrices [29] has considered the problem of receding horizon control for state-delayed systems where the delay is fixed. Also, they have presented an eigenvalue search algorithm to check the closed-loop stability of the system. However their control strategy is delay-independent, therefore highly conservative. Reference [30] introduced a novel optimization method for MPC of uncertain time-delay systems having constant state delays and constraint input. But, again, their method does not depend on the size of delay. A robust one-step LMI based MPC scheme is developed for discrete time-delay systems having fixed delay with polytopic-type uncertainty in [31]. Their proposed MPC method is obtained by minimizing a new cost function that includes multiterminal weighting terms, subject to constraints on input.

The deficiency of MPC systems in disturbance attenuation has drawn the attention towards the moving horizon \mathcal{H}_∞ control (MHHC) scheme in recent years, [26, 32, 33]. In this perspective, a Riccati-based MHHC algorithm was developed for a nominal time-delay system by [29, 34]. Then, a similar approach was proposed in [35] based on LMIs. However, all the abovementioned approaches are based on criteria which do not depend on the size of delay and therefore provide more conservative results. While taking the time-delay bounds into consideration, a delay-dependent MPC is proposed by [31] which guarantees only the closed-loop stability. Reference [35] has dealt with the receding horizon \mathcal{H}_∞ control for constraint time-delay systems having fixed delays. However, their method is delay-independent as the ones indicated above. Finally, proposing a new cost function for a finite horizon dynamic game problem which includes two terminal weighting terms and a delay-independent LMI condition, [36] has studied the receding horizon \mathcal{H}_∞ control problem for time-delay systems with fixed state delays.

Among these aforementioned work, it is apparently seen that only very few results exist concerning the MPC of TDSs which gives us a motivation to study this problem. Furthermore, to the best of author's knowledge, there does not exist any other reference in literature which deals with the robust \mathcal{H}_∞ MPC of uncertain time-delay systems having

time-varying delays in delay-dependent fashion. Therefore, combining these two sources of inspiration leads us to study that particular subject thoroughly.

In this paper, we investigate the design of a stabilizing \mathcal{H}_∞ state-feedback controller for linear state-delayed interval TDS having norm-bounded uncertainties and constrained inputs. First, based on the selection of an L - K functional, a stabilizing delay-dependent \mathcal{H}_∞ controller is introduced for nominal TDS having delay in the state, similar to the previous study of authors [37]. Then, the existing result is extended to cover TDS having norm-bounded uncertainties. Finally, the proposed approach is adapted to the so-called moving horizon scheme introduced in [26], to obtain a robust, delay-dependent moving horizon \mathcal{H}_∞ control technique which ensures a dissipative closed-loop system. The proposed technique is practically implementable since it takes the input constraints into consideration and it does not employ any linearization techniques such as cone-complementary method which is widely used in delay-dependent control approaches such as those used in [11].

The rest of this paper is organized as follows: Section 2 states the problem formulation. Mathematical background is given in Section 3 in order to provide a priori knowledge about the methodology followed. Section 4 is devoted to the proposed method. Two different illustrative examples are presented in Section 5 to demonstrate the effectiveness of the approach. Finally, Section 6 concludes the paper.

Notation. \mathbb{R} and \mathbb{R}^n represent the set of real numbers and of n -dimensional real vectors, respectively. Identity matrix with an appropriate dimension is denoted by I . $X > 0$ ($\geq, < 0$) implies that X is a positive-definite (positive semidefinite, negative-definite) matrix. Likewise, $X > Y$ means that $X - Y$ is positive definite. $(\cdot)^T$ indicates transpose of a real matrix. To avoid repetition, “ $*$ ” denotes off-diagonal block completion of a symmetric matrix. x_k and $x(k)$ are used interchangeably to simplify the notation especially in long mathematical expressions. Again, in order to make the notation easy, we drop the time-dependence of functions when it does not make any trouble. Finally, $\|x\|$ stands for the standard Euclidean 2-norm of the vector x . $\|x\|_{\max}$ is the absolute value of the entry of a vector x having the largest magnitude.

2. Problem Formulation

Let us consider a class of uncertain linear discrete-time, time-delay system of the form

$$\begin{aligned} x(k+1) &= A(k)x(k) + A_d(k)x(k-d(k)) \\ &\quad + B_w(k)w(k) + B_u(k)u(k), \\ z(k) &= Cx(k) + C_d x(k-d(k)) + D_w w(k) + D_u u(k), \\ x(k) &= \phi(k), \quad k \in [-d_{\max}, 0] \end{aligned} \quad (1)$$

subject to control constraints

$$\|u_i(k)\|_{\max} \leq u_{i,\max}^2, \quad \forall k \geq 0, \quad i = 1, 2, \dots, m, \quad (2)$$

where k stands for the k th sample-time, $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^m$ represents the control input, $u_{i,\max}$

is a known magnitude bound on the control effort in the i th channel, $\phi(k)$ is a given initial condition sequence, and $w(k) \in \mathbb{R}^q$ is a disturbance signal in l_2 , which satisfies

$$\sum_{i=0}^{\infty} \|w(i)\|^2 \leq \alpha^2. \quad (3)$$

$z(k) \in \mathbb{R}^p$ is the controlled output and $d(k) \in \mathbb{R}$ represents a time-varying delay which satisfies

$$d_{\min} \leq d(k) \leq d_{\max} \quad \forall k \geq 0, \quad (4)$$

where the nonnegative integers d_{\min} and d_{\max} stand for the lower and upper bounds of the delay $d(k)$, respectively, and all are assumed to be known. The time-varying delay $d(k)$ reduces to a constant delay d when $d_{\min} = d_{\max} = d$. On the other hand, the uncertain system matrices are assumed to be in the form of $A(k) = A + \Delta A(k)$, $A_d(k) = A_d + \Delta A_d(k)$, and $B_u(k) = B_u + \Delta B_u(k)$. Here, the unknown matrices $\Delta A(k)$, $\Delta A_d(k)$, and $\Delta B_u(k)$ are real-valued time-varying matrix functions representing the parameter uncertainties of the system and are in the form of

$$[\Delta A(k) \quad \Delta A_d(k) \quad \Delta B_u(k)] = GF(k) [E_1 \quad E_2 \quad E_3], \quad (5)$$

where G, E_1, E_2 are known time-invariant matrices and $F(k)$ is an unknown time-varying matrix function with Lebesgue measurable elements satisfying

$$F(k)^T F(k) \leq I, \quad \forall k \geq 0. \quad (6)$$

The objective of infinite horizon MHHC problem is to obtain the state-feedback control law in the form of $u(k) = K_k x(k)$ at each instant k , so that the following conditions hold for the closed-loop system.

(I) The closed-loop system

$$\begin{aligned} x(k+1) &= A_{cl}(k)x(k) + A_d(k)x(k-d(k)) + B_w w(k), \\ z(k) &= C_{cl}x(k) + C_d x(k-d(k)) + D_w w(k), \\ x(k) &= 0, \quad k \in [-d_{\max}, 0], \end{aligned} \quad (7)$$

where $A_{cl}(k) \triangleq A(k) + B_u(k)K_k$ and $C_{cl} \triangleq C(k) + D_u(k)K_k$ is globally uniformly asymptotically stable under the conditions $w(k) = 0$ for all $k \geq 0$.

(II) Consider

$$\|z(k)\|^2 \leq \gamma^2 \|w(k)\|^2 \quad \forall k \geq 0 \quad (8)$$

and for all nonzero disturbance signal $w(k) \in l_2[0, \infty)$ and a given scalar $\gamma > 0$ under the condition $x(k) = 0$ for all $k \in [-d_{\max}, 0]$.

(III) Let us define two nested ellipsoids as follows:

$$\varepsilon_1(P, Q, r) := x \in \mathbb{R}^n : V(k) \leq r \quad \forall k \geq 0 \quad (9)$$

$$\varepsilon_2(P, Q, r, \alpha) := x \in \mathbb{R}^n : \gamma^2 \alpha^2 + V(k) \leq r \quad \forall k \geq 0,$$

where $V(k)$ is the Lyapunov function. If the initial state $x(0)$ satisfies $\gamma^2 \alpha^2 + V(0) \leq r$, that is, $x(0)$ belongs to ε_2 , then all perturbed state trajectories remain in the ellipsoid ε_1 .

(IV) The constraint on the size of control effort (2) is satisfied for all $k \geq 0$.

In order to avoid complexity of the main result section, some related and required background material are provided next.

3. Preliminaries

We start with the definition of *dissipativity* which plays an important role during the implementation of moving horizon control.

Definition 1 (dissipativity). The system (7) with supply rate $s(w(k), z(k)) \triangleq \gamma^2 \|w(k)\|^2 - \|z(k)\|^2$ is said to be dissipative if there exists a quadratic storage function $V(x(k))$ such that

$$V(k) + s(w(k), z(k)) \geq V(k+1) \quad \forall k \geq 0. \quad (10)$$

This implies that the change of storage function from step k to step $(k+1)$ is always less than the supplied rate to the system. Therefore, inequality (10) is named as dissipation inequality. If the dissipation inequality (10) holds true for any system trajectory of (1) and (4), then for any disturbance signal satisfying (3), the energy of the controlled output satisfies

$$\sum_{i=0}^{\infty} \|z(i)\|^2 \leq r, \quad (11)$$

where r is an upper bound for output energy and l_2 gain of the closed-loop system (7) from the disturbance $w(k)$ to the performance output $z(k)$ is bounded by γ .

4. Main Results

Taking $A(k) = A$, $A_d(k) = A_d$, and $B_u(k) = B_u$ in (1), the following theorem describes a criterion for constructing an \mathcal{H}_{∞} state-feedback controller in the form $u(k) = Kx(k)$, such that the closed-loop nominal system (7) with (4) is globally asymptotically stable.

Theorem 2. Given the scalar constants d_{\min} , d_{\max} , and γ , the state-feedback control law $u(k) = LY^{-1}x(k)$ globally asymptotically stabilizes the time-delayed system (1) with (4) with an \mathcal{H}_{∞} disturbance attenuation level, γ , if there exist matrices $Y = Y^T > 0$, $W = W^T > 0$, and L with appropriate dimensions satisfying

$$\begin{bmatrix} Y & * & * & * & * & * \\ 0 & W & * & * & * & * \\ 0 & 0 & \gamma^2 I & * & * & * \\ AY + B_u L & A_d W & B_w & Y & * & * \\ CY + D_u L & C_d W & D_w & 0 & I & * \\ Y & 0 & 0 & 0 & 0 & d_m^{-1} W \end{bmatrix} > 0, \quad (12)$$

where $d_m \triangleq d_{\max} - d_{\min} + 1$.

Proof. Let us choose a candidate L - K functional as follows:

$$V(k) = V_1(k) + V_2(k) + V_3(k), \quad (13)$$

where

$$\begin{aligned} V_1(k) &= x^T(k) P x(k), \\ V_2(k) &= \sum_{s=k-d(k)}^{k-1} x^T(s) Q x(s), \\ V_3(k) &= \sum_{i=-d_{\max}+2}^{-d_{\min}+1} \sum_{s=k+i-1}^{k-1} x^T(s) Q x(s). \end{aligned} \quad (14)$$

In view of the closed-loop system trajectory (7), one can define a first difference of the energy function $V(k)$ as follows:

$$\Delta V(k) \triangleq V(k+1) - V(k) = \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k), \quad (15)$$

where

$$\begin{aligned} \Delta V_1(k) &= x^T(k+1) P x(k+1) - x^T(k) P x(k) \\ &= x^T(k) (A_{\text{cl}}^T P A_{\text{cl}} - P) x(k) \\ &\quad + 2x^T(k) A_{\text{cl}}^T P A_d x(k-d(k)) \\ &\quad + 2x^T(k) A_{\text{cl}}^T P B_w w(k) \\ &\quad + x^T(k-d(k)) A_d^T P A_d x(k-d(k)) \\ &\quad + 2x^T(k-d(k)) A_d^T P B_w w(k) \\ &\quad + w^T(k) B_w^T P B_w w(k), \\ \Delta V_2(k) &= \sum_{s=k-d(k+1)+1}^k x^T(s) Q x(s) - \sum_{s=k-d(k)}^{k-1} x^T(s) Q x(s) \\ &= x^T(k) Q x(k) - x^T(k-d(k)) Q x(k-d(k)) \\ &\quad + \sum_{s=k-d(k+1)+1}^{k-1} x^T(s) Q x(s) - \sum_{s=k-d(k)+1}^{k-1} x^T(s) Q x(s). \end{aligned} \quad (16)$$

Since

$$\begin{aligned} &\sum_{s=k-d(k+1)+1}^k x^T(s) Q x(s) \\ &= \sum_{s=k-d_{\min}+1}^k x^T(s) Q x(s) + \sum_{s=k-d(k+1)+1}^{k-d_{\min}} x^T(s) Q x(s) \\ &\leq \sum_{s=k-d(k)+1}^k x^T(s) Q x(s) + \sum_{s=k-d_{\max}+1}^{k-d_{\min}} x^T(s) Q x(s) \end{aligned} \quad (18)$$

we can write

$$\begin{aligned} \Delta V_2(k) &\leq x^T(k) Q x(k) - x^T(k-d(k)) Q x(k-d(k)) \\ &\quad + \sum_{s=k-d_{\max}+1}^{k-d_{\min}} x^T(s) Q x(s). \end{aligned} \quad (19)$$

Finally,

$$\begin{aligned} \Delta V_3(k) &= \sum_{i=-d_{\max}+2}^{-d_{\min}+1} \left(\sum_{s=k+i}^k x^T(s) Q x(s) - \sum_{s=k+i-1}^{k-1} x^T(s) Q x(s) \right) \\ &= \sum_{i=-d_{\max}+2}^{-d_{\min}+1} (x^T(k) Q x(k) - x^T(k+i-1) Q x(k+i-1)) \\ &= (d_{\max} - d_{\min}) x^T(k) Q x(k) - \sum_{i=k-d_{\max}+1}^{k-d_{\min}} x^T(i) Q x(i). \end{aligned} \quad (20)$$

Then, in the light of (16), (19), and (20), a bound on $\Delta V(k)$ can be obtained as follows:

$$\begin{aligned} \Delta V(k) &\leq x^T(k) (A_{\text{cl}}^T P A_{\text{cl}} - P) x(k) \\ &\quad + 2x^T(k) A_{\text{cl}}^T P A_d x(k-d(k)) \\ &\quad + 2x^T(k) A_{\text{cl}}^T P B_w w(k) \\ &\quad + x^T(k-d(k)) A_d^T P A_d x(k-d(k)) \\ &\quad + 2x^T(k-d(k)) A_d^T P B_w w(k) \\ &\quad + w^T(k) B_w^T P B_w w(k) \\ &\quad + x^T(k) Q x(k) - x^T(k-d(k)) Q x(k-d(k)) \\ &\quad + \sum_{s=k-d_{\max}+1}^{k-d_{\min}} x^T(s) Q x(s) \\ &\quad + (d_{\max} - d_{\min}) x^T(k) Q x(k) \\ &\quad - \sum_{s=k-d_{\max}+1}^{k-d_{\min}} x^T(s) Q x(s). \end{aligned} \quad (21)$$

Then, if we define the right-hand side of (21) with $\bar{\Delta V}(k)$, it is obvious that

$$\begin{aligned} \Delta V(k) &+ z^T(k) z(k) - \gamma^2 w^T(k) w(k) \\ &\leq \bar{\Delta V}(k) + z^T(k) z(k) - \gamma^2 w^T(k) w(k). \end{aligned} \quad (22)$$

Therefore, if

$$\bar{\Delta V}(k) + z^T(k) z(k) - \gamma^2 w^T(k) w(k) < 0 \quad (23)$$

is satisfied for all $k \geq 0$, then the nominal closed-loop system (7) with (4) is guaranteed to be globally, uniformly, and asymptotically stable when $w(k) = 0$ for all $k \geq 0$, and we get $\|z\|_2^2 \leq \gamma^2 \|w\|_2^2$ for all $w(k) \in l_2[0, \infty)$. Equivalently, a straightforward application of the Schur complement formula on (23) allows us to write

$$\begin{bmatrix} d_m Q - P & * & * & * & * \\ 0 & -Q & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ A_{cl} & A_d & B_w & -P^{-1} & * \\ C_{cl} & C_d & D_w & 0 & -I \end{bmatrix} < 0, \quad (24)$$

where $d_m := d_{\max} - d_{\min} + 1$. Then, substituting $A_{cl} = A + B_u K$ and $C_{cl} = C + D_u K$ into (24), pre- and postmultiplying (24)

$$\begin{bmatrix} Y & * & * & * & * & * & * \\ 0 & W & * & * & * & * & * \\ 0 & 0 & \gamma^2 I & * & * & * & * \\ AY + B_u L & A_d W & B_w & Y - \epsilon G G^T & * & * & * \\ CY + D_u L & C_d W & D_w & 0 & I & * & * \\ Y & 0 & 0 & 0 & 0 & d_m^{-1} W & * \\ E_2 L + E_1 Y & E_3 W & 0 & 0 & 0 & 0 & \epsilon I \end{bmatrix} > 0. \quad (25)$$

Proof. Replacing A , A_d , and B_u with $A(k) = A + \Delta A$, $A_d(k) = A_d + \Delta A_d$, and $B_u(k) = B_u + \Delta B_u$ in (12) and utilizing the definitions given in (5) and (6) together with the definitions

$$\begin{aligned} \Pi &\triangleq [E_2 L + E_1 Y \quad E_3 W \quad 0 \quad 0 \quad 0 \quad 0], \\ \Theta &\triangleq [0 \quad 0 \quad 0 \quad G^T \quad 0 \quad 0]^T \end{aligned} \quad (26)$$

yield

$$\begin{bmatrix} -Y & * & * & * & * & * \\ 0 & -W & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * \\ AY + B_u L & A_d W & B_w & -Y & * & * \\ CY + D_u L & C_d W & D_w & 0 & -I & * \\ Y & 0 & 0 & 0 & 0 & -d_m^{-1} W \end{bmatrix} + \Pi^T F(k) \Theta^T + \Theta F(k)^T \Pi < 0. \quad (27)$$

Note that for any matrices Σ_1 and Σ_2 of appropriate dimensions, there always exists an $\epsilon > 0$ such that

$$\Sigma_1^T \Sigma_2 + \Sigma_2^T \Sigma_1 < \frac{1}{\epsilon} \Sigma_1^T \Sigma_1 + \epsilon \Sigma_2^T \Sigma_2. \quad (28)$$

by $\text{diag}\{P^{-1}, Q^{-1}, I, I, I\}$, and applying the Schur complement which is followed by defining $Y := P^{-1}$, $W := Q^{-1}$ immediately lead to the LMI condition given in (12). This concludes our proof. \square

Now, let us consider (1) with uncertainties. The following corollary considers the robust state-feedback \mathcal{H}_∞ controller design problem for the uncertain linear time-delay system given in (1), (4) with (5).

Corollary 3. *The control law $u(k) = LY^{-1}x(k)$ robustly asymptotically stabilizes the uncertain system (1) and guarantees the \mathcal{H}_∞ performance index, γ , for any time-varying delay, $d(k)$, satisfying (4), if there exist matrices $Y = Y^T > 0$, $W = W^T > 0$, and L in appropriate dimensions and a scalar $\epsilon > 0$ which satisfy the following LMI condition:*

Therefore

$$\Pi^T F(k) \Theta^T + \Theta F(k)^T \Pi \leq \frac{1}{\epsilon} \Pi^T \Pi + \epsilon \Theta \Theta^T. \quad (29)$$

Then, if

$$\begin{bmatrix} -Y & * & * & * & * & * \\ 0 & -W & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * \\ AY + B_u L & A_d W & B_w & -Y & * & * \\ CY + D_u L & C_d W & D_w & 0 & -I & * \\ Y & 0 & 0 & 0 & 0 & -d_m^{-1} W \end{bmatrix} + \frac{1}{\epsilon} \Pi^T \Pi + \epsilon \Theta \Theta^T < 0 \quad (30)$$

is satisfied, (27) is also satisfied. Finally, applying Schur complement formula to (30) yields (25). This completes the proof. \square

Next we will introduce some LMI conditions which fulfill the requirement given in item (III).

Lemma 4. *Suppose that there exists a solution (γ, Y, W, X) that meets the stability condition (25) with input constraint (2) for system (1) and (4), then state trajectory starting from $x(0)$ remains in $\epsilon_1(P, r)$, if there exist matrices $P = P^T > 0$ and $Q = Q^T > 0$ for given positive scalar constants α and r , such that*

$$\begin{bmatrix} r - \gamma^2 \alpha^2 & * & * & \cdots & * & \cdots & * & * & * \\ x(k) & Y & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 \\ x(k-1) & 0 & d_m^{-1}W & \cdots & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots \\ x(k-d_{\min}) & 0 & 0 & \cdots & d_m^{-1}W & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots \\ x(k-d_{\max}+2) & 0 & 0 & \cdots & 0 & \cdots & 3^{-1}W & 0 & 0 \\ x(k-d_{\max}+1) & 0 & 0 & \cdots & 0 & \cdots & 0 & 2^{-1}W & 0 \\ x(k-d_{\max}) & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & W \end{bmatrix} \geq 0, \quad (31)$$

where $d_m = (d_{\max} - d_{\min} + 1)$.

Proof. The LMI condition (31) corresponds to the state constraint that is defined in (9). Utilizing the energy bound of disturbance given in (3), the dissipation inequality (10) implies

$$V(x(k)) + \sum_{i=0}^{k-1} \|z(i)\|^2 \leq V(x(0)) + \gamma^2 \alpha^2 \quad \forall k \geq 0. \quad (32)$$

Therefore, the output energy is bounded as (11). As explained in [27],

- (i) if $r > 0$, $x(k) \in \varepsilon_1(P, Q, r)$ for all $k \geq 0$;
- (ii) if $r > \gamma^2 \alpha^2$, $x(k) \in \varepsilon_1(P, Q, r)$ for $x(0) \in \varepsilon_2(P, r, \alpha)$.

Here, by choosing $r > \gamma^2 \alpha^2$, the state trajectory starting from $x(0)$ meets $\sum_{i=0}^{\infty} \|z(i)\|^2 \leq r$ and $V(x(k)) \leq r$ if $V(x(k)) \leq r - \gamma^2 \alpha^2$ with respect to (32) as mentioned in (8). Note that it is easy to see that the LMI condition (31) is congruent to

$$\begin{aligned} x^T(k) P x(k) + \sum_{s=k-d(k)}^{k-1} x^T(s) Q x(s) \\ + \sum_{i=-d_{\max}+2}^{-d_{\min}+1} \sum_{s=k+i-1}^{k-1} x^T(s) Q x(s) \leq r - \gamma^2 \alpha^2 \end{aligned} \quad (33)$$

in view of the variable changes $Y := P^{-1}$ and $W := Q^{-1}$ and the successive implementations of Schur complement formulae together with the bounding

$$\begin{aligned} \sum_{s=k-d(k)}^{k-1} x^T(s) Q x(s) &\leq \sum_{s=k-d_{\min}}^{k-1} x^T(s) Q x(s) \\ &+ \sum_{s=k-d_{\max}+1}^{k-d_{\min}-1} x^T(s) Q x(s), \end{aligned} \quad (34)$$

which has been used to eliminate the requirement on the knowledge of $d(k)$. \square

On the other hand, we need to take the constrained input into consideration to fulfill the requirement given in item (IV) of the problem statement. Therefore, regarding the state trajectories belonging to ε_1 , the control constraint condition given in (2) is satisfied as follows [38]:

$$\begin{aligned} \max_{k \geq 0} |u_i(k)|^2 &= \max_{k \geq 0} |(LY^{-1})_i x(k)|^2 \\ &\leq \max_{x \in \varepsilon_1(P, r)} |(LY^{-1})_i x|^2 \\ &\leq r \|(LY^{-1/2})_i x\|^2 = r (LY^{-1} L^T)_{ii}. \end{aligned} \quad (35)$$

Hence, taking the Schur complement of (35) allows us to obtain the following LMI condition which is equivalent to the constraint (2):

$$\begin{bmatrix} \frac{1}{rX} & L \\ L^T & Y \end{bmatrix} \geq 0, \quad X_{ii} \leq u_{i,\max}^2 \quad (36)$$

for some $X = X^T > 0$.

Therefore, for given scalar constants $\alpha > 0$ and $r > 0$, the optimization problem can be stated as follows:

$$\begin{aligned} \min_{L, Y, W, X} \quad & \gamma^2 \\ \text{subject to} \quad & (22) \ (26), \ (31). \end{aligned} \quad (37)$$

Next, we will extend the \mathcal{H}_∞ controller design approach to the moving horizon scheme. Exploiting the moving horizon strategy, one could solve the optimization problem given in (37) by utilizing the actual state measurements provided at each time instant k . Unfortunately, this simple implementation of the moving horizon strategy might fail to guarantee

the dissipation requirement given in the statement of the problem (see [39] for details). For that reason, in order to extend the idea of \mathcal{H}_∞ control approach provided for the TDSs to the moving horizon platform, one needs to introduce some additional convex requirement so that the dissipativity of the closed-loop system is ensured. Next theorem provides an LMI condition for the optimization problem so that the closed-loop system is guaranteed to be dissipative.

Theorem 5. Suppose that there exists an optimal solution $(\gamma_0, Y_0, W_0, X_0)$ that meets the LMIs (25), (31), and (36) for system (1) that is controlled with $u = K_0 x$, where $K_0 = L_0 Y_0^{-1}$. Then the dissipation inequality

$$\begin{aligned} & \sum_{i=0}^k (\|z(i)\|^2 - \gamma^2 \|w(i)\|^2) \\ & \leq x(0)^T P_0 x(0) + \sum_{s=-d_0}^{-1} x(s)^T Q_0 x(s) \\ & \quad + \sum_{j=-d_{\max}+2}^{-d_{\min}+1} \sum_{s=j-1}^{-1} x(s)^T Q_0 x(s) \end{aligned} \quad (38)$$

also holds if the LMI condition

$$\begin{bmatrix} \Omega & * & * & \cdots & * & \cdots & * & * \\ x(k) & Y_k & 0 & \cdots & 0 & \cdots & 0 & 0 \\ x(k-1) & 0 & d_m^{-1} W_k & \cdots & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ x(k-d_{\min}) & 0 & 0 & \cdots & d_m^{-1} W_k & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ x(k-d_{\max}+2) & 0 & 0 & \cdots & 0 & \cdots & 2^{-1} W_k & 0 \\ x(k-d_{\max}+1) & 0 & 0 & \cdots & 0 & \cdots & 0 & W_k \end{bmatrix} \geq 0, \quad (39)$$

where $d_m = (d_{\max} - d_{\min} + 1)$ and

$$\begin{aligned} \Omega = & p_0 + q_0 - p_{k-1} - q_{k-1} + x^T(k) P_{k-1} x(k) \\ & + \sum_{s=k-d_{\max}}^{k-1} x^T(s) Q_{k-1} x(s) \\ & + \sum_{j=-d_{\max}+2}^{-d_{\min}+1} \sum_{s=j+k-1}^{k-1} x^T(s) Q_{k-1} x(s). \end{aligned} \quad (40)$$

with

$$\begin{aligned} p_k = & p_{k-1} - x(k)^T [P_{k-1} - P_k] x(k) \\ q_k = & q_{k-1} - \sum_{s=k-d_{\min}}^{k-1} x(s)^T Q_{k-1} x(s) - \sum_{k-d_{\min}}^{k-1} x(s)^T Q_k x(s) \\ & - \sum_{j=-d_{\max}+2}^{-d_{\min}+1} \sum_{s=j+k-1}^{k-1} x(s)^T [Q_{k-1} - Q_k] x(s) \end{aligned} \quad (41)$$

holds true when Y_k and W_k are replaced with $Y_0 = Y_0^T$ and $W_0 = W_0^T$, respectively.

Proof. Assume there exists a feasible solution set which holds (39). Let us consider the predetermined Lyapunov-Krasovskii

candidate functional for the i th energy level as follows:

$$\begin{aligned} V_i = & x^T(i) P x(i) + \sum_{s=i-d(i)}^{i-1} x^T(s) Q x(s) \\ & + \sum_{j=-d_{\max}+2}^{-d_{\min}+1} \sum_{s=i+j-1}^{i-1} x^T(s) Q x(s). \end{aligned} \quad (42)$$

By rearranging inequality (10) at each step i and assuming that the applied control at i th instant is kept constant up to the $(i+1)$ th instant, one can write the dissipation inequality as

$$V_{i+1} + \|z(i)\|^2 \leq \gamma^2 \|w(i)\|^2 + V_i \quad (43)$$

for $i = 0, 1, \dots, k$, where V_{i+1} is the energy of the closed-loop system attained by using the Lyapunov matrices obtained at the i th step. Then we get

$$\begin{aligned} & \sum_{i=0}^k (\|z(i)\|^2 - \gamma^2 \|w(i)\|^2) \\ & \leq x(0)^T P_0 x(0) - x(k+1)^T P_k x(k+1) \\ & \quad - \sum_{i=1}^k x(i)^T [P_{i-1} - P_i] x(i) + \sum_{s=-d_0}^{-1} x(s)^T Q_0 x(s) \end{aligned}$$

$$\begin{aligned}
& - \sum_{s=-d_{k+1}+k+1}^k x(s)^T Q_k x(s) \\
& - \sum_{i=1}^k \sum_{s=i-d_i}^{i-1} x(s)^T [Q_{i-1} - Q_i] x(s) \\
& + \sum_{j=-d_{\max+2}}^{-d_{\min}+1} \sum_{s=j-1}^{-1} x(s)^T Q_0 x(s) \\
& - \sum_{j=-d_{\max+2}}^{-d_{\min}+1} \sum_{s=j+k}^k x(s)^T Q_k x(s) \\
& - \sum_{i=1}^k \sum_{j=-d_{\max+2}}^{-d_{\min}+1} \sum_{s=j+i-1}^{i-1} x(s)^T [Q_{i-1} - Q_i] x(s).
\end{aligned} \tag{44}$$

Assuming that $P_k \geq 0, Q_k \geq 0$ for all $k \geq 0$, one can write

$$\begin{aligned}
& \sum_{i=0}^k (\|z(i)\|^2 - \gamma^2 \|w(i)\|^2) \\
& \leq x(0)^T P_0 x(0) - \sum_{i=1}^k x(i)^T [P_{i-1} - P_i] x(i) \\
& + \sum_{s=-d_0}^{-1} x(s)^T Q_0 x(s) \\
& - \sum_{i=1}^k \sum_{s=i-d_i}^{i-1} x(s)^T [Q_{i-1} - Q_i] x(s) \\
& + \sum_{j=-d_{\max+2}}^{-d_{\min}+1} \sum_{s=j-1}^{-1} x(s)^T Q_0 x(s) \\
& - \sum_{i=1}^k \sum_{j=-d_{\max+2}}^{-d_{\min}+1} \sum_{s=j+i-1}^{i-1} x(s)^T [Q_{i-1} - Q_i] x(s).
\end{aligned} \tag{45}$$

Note that if

$$\begin{aligned}
& \sum_{i=1}^k x(i)^T (P_{i-1} - P_i) x(i) + \sum_{i=1}^k \sum_{s=i-d_i}^{i-1} x(s)^T [Q_{i-1} - Q_i] x(s) \\
& + \sum_{i=1}^k \sum_{j=-d_{\max+2}}^{-d_{\min}+1} \sum_{s=j+i-1}^{i-1} x(s)^T [Q_{i-1} - Q_i] x(s) \geq 0
\end{aligned} \tag{46}$$

then the closed-loop system behaves dissipatively. However, inequality (46) involves the term

$$\sum_{i=1}^k \sum_{s=i-d_i}^{i-1} x(s)^T [Q_{i-1} - Q_i] x(s) \tag{47}$$

which is dependent on the knowledge of $d(i)$. Therefore, if one can find a lower bound, B , such that

$$\sum_{i=1}^k \sum_{s=i-d_i}^{i-1} x(s)^T [Q_{i-1} - Q_i] x(s) \geq B \geq 0 \tag{48}$$

then we guarantee the dissipativity of the overall closed-loop system. Note that

$$\begin{aligned}
& \sum_{i=1}^k \sum_{s=i-d_i}^{i-1} x(s)^T [Q_{i-1} - Q_i] x(s) \\
& \geq \sum_{i=1}^k \sum_{s=i-d_{\min}}^{i-1} x(s)^T [Q_{i-1} - Q_i] x(s).
\end{aligned} \tag{49}$$

Hence, we need to satisfy

$$\begin{aligned}
& \sum_{i=1}^k x(i)^T (P_{i-1} - P_i) x(i) + \sum_{i=1}^k \sum_{s=i-d_{\min}}^{i-1} x(s)^T [Q_{i-1} - Q_i] x(s) \\
& + \sum_{i=1}^k \sum_{j=-d_{\max+2}}^{-d_{\min}+1} \sum_{s=j+i-1}^{i-1} x(s)^T [Q_{i-1} - Q_i] x(s) \geq 0.
\end{aligned} \tag{50}$$

We infer from (41) that

$$\begin{aligned}
p_{k-1} &= p_0 - \sum_{i=1}^{k-1} x(i)^T [P_{i-1} - P_i] x(i) \\
q_{k-1} &= q_0 - \sum_{i=1}^k \sum_{s=i-d_i}^{i-1} x(s)^T [Q_{i-1} - Q_i] x(s) \\
& - \sum_{i=1}^k \sum_{j=-d_{\max+2}}^{-d_{\min}+1} \sum_{s=j+i-1}^{i-1} x(s)^T [Q_{i-1} - Q_i] x(s).
\end{aligned} \tag{51}$$

Applying Schur complement formula to LMI (39) yields

$$\begin{aligned}
& p_0 - p_{k-1} + x(k)^T [P_{k-1} - P_k] x(k) + q_0 - q_{k-1} \\
& + \sum_{s=k-d_{\min}}^{k-1} x(s)^T [Q_{k-1} - Q_k] x(s) \\
& + \sum_{j=-d_{\max+2}}^{-d_{\min}+1} \sum_{s=j+k-1}^{k-1} x(s)^T [Q_{k-1} - Q_k] x(s) \geq 0.
\end{aligned} \tag{52}$$

Then, using the definitions given in (51) and in (52) yields (50). This concludes the proof. \square

Therefore, in view of (36) together with Corollary 3, Lemma 4, and Theorem 5, we present a new convex optimization problem as follows.

Theorem 6. Given scalar constants $\alpha_0 > 0$ and $r_0 > 0$, at each step k , if one can find a common feasible solution set $\{\gamma_k, L_k, Y_k, W_k, X_k\}$ to the convex optimization problem

$$\begin{aligned}
& \min_{L, Y, W, X} \gamma^2 \\
& \text{subject to } (25), (31), (36), (39)
\end{aligned} \tag{53}$$

then system (1) controlled with state-feedback control law $u(k) = K_k x(k)$, where $K_k = L_k Y_k^{-1}$, has the following properties.

- (i) The closed-loop system is \mathcal{H}_∞ stable.
- (ii) The \mathcal{H}_∞ gain γ from disturbance input w to controlled output z is minimum.
- (iii) The control input constraint is satisfied.
- (iv) The closed-loop system is dissipative.

The following receding-horizon \mathcal{H}_∞ algorithm can be used to find the optimal control law for system (1) with (4) which repeats at each time instant k .

Algorithm 7.

Step 1. Defining the step size for γ as $d\gamma$; first, fix γ to a sufficiently large value and solve the optimization problem (37) for given α_0 , r_0 , d_{\max} , and d_{\min} . If there exists a feasible solution to the problem, set $\gamma = \gamma - d\gamma$ until infeasibility. Then, assign $\gamma_{\text{opt}} = \gamma + d\gamma$. Compute P_0 , Q_0 and $K_0 = L_0 Y_0^{-1}$ and the initial dissipation level is $p(0) = x(0)^T P_0 x(0)$ and $q(0) = \sum_{s=-d_{\min}}^{-1} x^T(s) Q_0 x(s) + \sum_{s=-d_{\max}+2}^{-d_{\min}+1} \sum_{j=1}^{-1} x^T(s) Q_0 x(s)$. Go to Step 3.

Step 2. According to given α_0 , r_0 , d_{\max} , and d_{\min} and obtained dissipation level with $p(k)$ and $q(k)$, solve the optimization problem (53). If there exists a feasible solution, set $\gamma(k) = \gamma(k) - d\gamma$, solve (53) until infeasibility. Then, assign $\gamma(k) \triangleq \gamma(k) + d\gamma$. If there is no possible solution increase r until a feasible solution is found and repeat the minimization procedure for $\gamma(k)$. Then, for γ_{\min} , compute P_k , Q_k and $K_k = L_k Y_k^{-1}$ and the dissipation level p_k and q_k . Go to Step 3.

Step 3. Apply the control signal $u(k) = K_k x(k)$ to the system. Change the step from k to $k + 1$ and continue with Step 2.

5. Numerical Examples

In this section we give two numerical examples to demonstrate the efficiency of the proposed method for time-varying state delayed systems with and without norm-bounded parametric uncertainties, respectively. Note that the results are obtained by using SeDuMi and YALMIP toolboxes operated under MATLAB.

Example 1. The following example is borrowed from [39] in which a nominal \mathcal{H}_∞ control problem has been considered. Consider a state-delayed system (1) whose model parameters are given by

$$A = \begin{bmatrix} 0 & 1 \\ -0.14 & 0.9 \end{bmatrix} \quad A_d = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix} \quad B_w = B_u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = [0.5 \quad 1] \quad C_d = [0 \quad 0] \quad D_w = [0] \quad D_u = [0.1]$$

$$x(i) = [0 \quad -1.5]^T \quad i = 0, -1, \dots, -d_{\max}$$

$$\|u(k)\| \leq 1, \quad \forall k \geq 0.$$

(54)

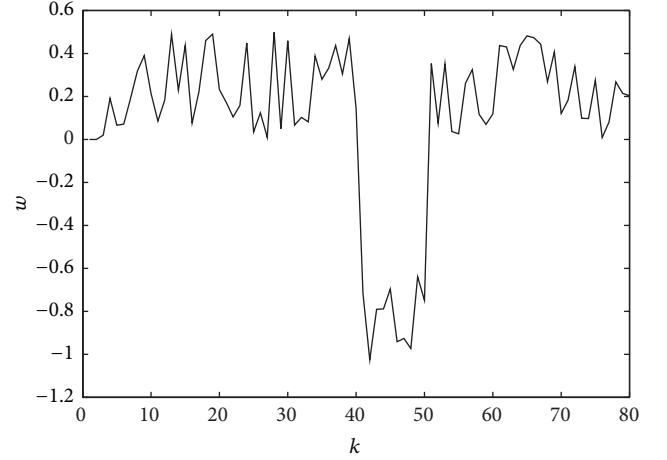


FIGURE 1: The time-history of the disturbance signal affecting the system.

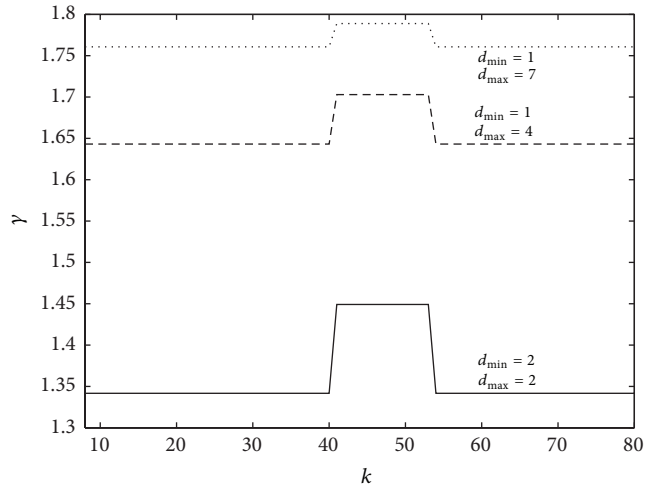


FIGURE 2: The variation of disturbance attenuation level, γ , for different delay sizes.

Choosing $r_0 = 0.17$ and using the energy-bounded disturbance ($\alpha = 0.3$), whose time-history is as shown in Figure 1, we compute $\gamma_{\text{opt}} = 1.09$ for $d_{\max} = d_{\min} = 2$ at the initial step of algorithm. Applying the proposed algorithm on the system, we obtain satisfactory results in γ minimization which also leads us to obtain a significant improvement in disturbance rejection performance. Figure 2 shows the variations of γ with respect to various delay bounds. It can be seen in the figure that the value of γ decreases while the bounds on the delay are tightened. Thus, the conservatism of the approach can be reduced by bounding the delay if the upper and lower bounds of delay are known. For different delay bounds, d_{\max} and d_{\min} , the variation of the controlled output, z , and the control signal, u , are shown in Figures 3, 4, and 5, proving that the results are satisfactory and the control input constraint is satisfied for the system that involves time-varying delay.

Besides, the simulations are repeated for the same example with a new A_d matrix determined in a way that $A + A_d$

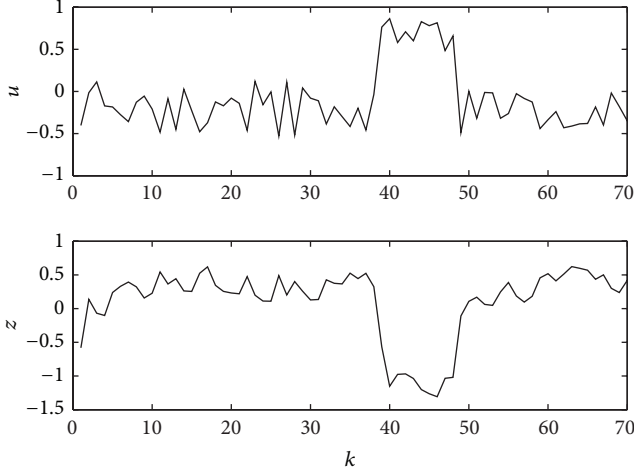


FIGURE 3: The time-history of the controlled output, z , and the control input, u , for $d_{\max} = d_{\min} = 2$.

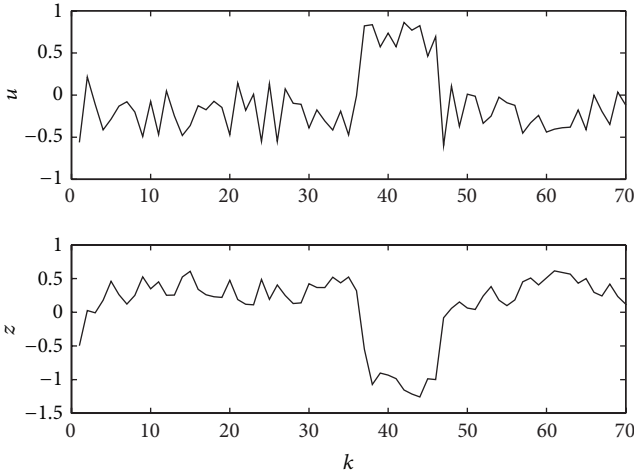


FIGURE 4: The time-history of the controlled output, z , and the control input, u , for $d_{\max} = 4, d_{\min} = 1$.

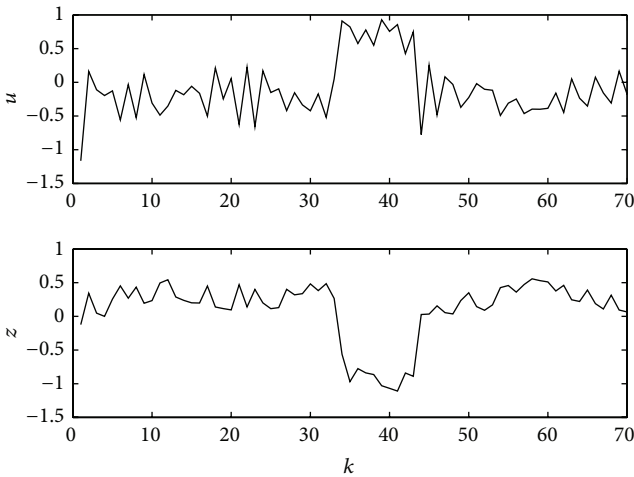


FIGURE 5: The time-history of the controlled output, z , and the control input, u , for $d_{\max} = 7, d_{\min} = 1$.

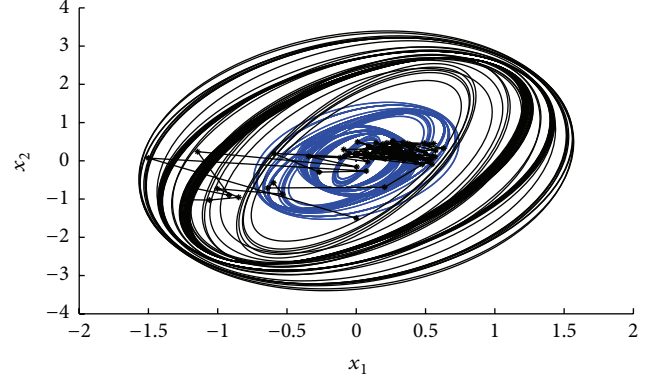


FIGURE 6: Ellipsoidal bounds on state trajectories, for $d_{\max} = 2, d_{\min} = 2$.

has a pole outside of the unit circle when the delay $d(k) = 0$. Consider the following:

$$A = \begin{bmatrix} 0 & 1 \\ -0.14 & 0.9 \end{bmatrix} \quad A_d = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}. \quad (55)$$

Hence, the effectiveness of the control algorithm is demonstrated also for unstable systems. The state trajectory that remains in the time-invariant ellipsoids is given in Figure 6.

Example 2. We utilize the example given in [40] in revised form as follows:

$$\begin{aligned} A &= \begin{bmatrix} 0 & 0.1 \\ -0.14 & 0.9 \end{bmatrix} & A_d &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix} \\ B_w = B_u &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} & C &= \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{0.1} \end{bmatrix} & C_d &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ D_w &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} & D_u &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{aligned} \quad (56)$$

$$x(i) = [0 \ 0]^T \quad i = 0, -1, \dots, -d_{\max}$$

$$\|u(k)\| \leq 1, \quad \forall k \geq 0,$$

where $\|A\| = \|A_d\| = \|B_u\| \leq 0.1$. The expansion of the proposed method to the time-delay systems having parametric uncertainties is shown in the example. The time-history of the disturbance applied to the system is as shown in Figure 1. We compute $\gamma_{\text{opt}} = 1.26$ for $d_{\max} = d_{\min} = 2$ and we choose $r_0 = 0.1$. During the simulations, the best variation of r is obtained at each step. Figure 7 shows the variations in γ with respect to different delay-bounds. For various delay bounds, d_{\max} and d_{\min} , we obtain the time-history of performance outputs and the control efforts as shown in Figures 8, 9, and 10. Besides, Figure 11 shows the invariant ellipsoidal bounds on the state trajectory of the delayed system.

6. Conclusion

The MPC of a class of linear discrete-time, uncertain time-delayed systems having time-varying interval delays was

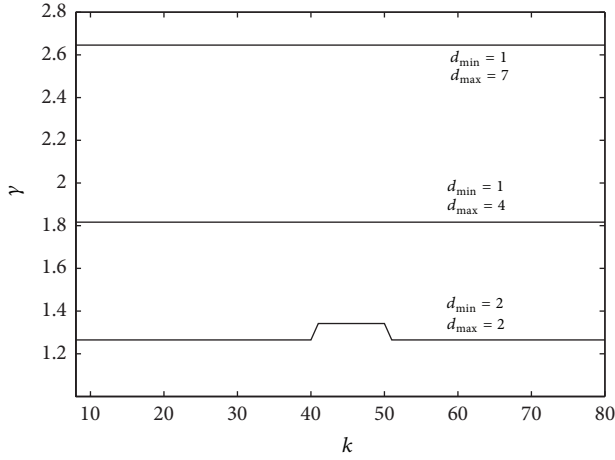


FIGURE 7: The variations of disturbance attenuation level, γ , for different delay sizes.

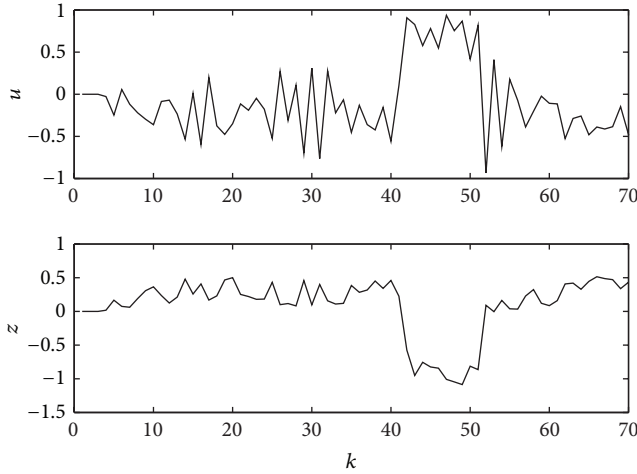


FIGURE 8: The time-history of the controlled output, z , and the control input, u , for $d_{\max} = 2$, $d_{\min} = 2$.

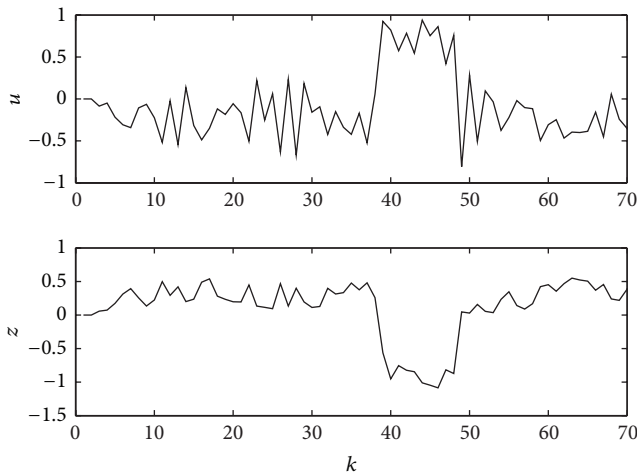


FIGURE 9: The time-history of the controlled output, z , and the control input, u , for $d_{\max} = 4$, $d_{\min} = 1$.

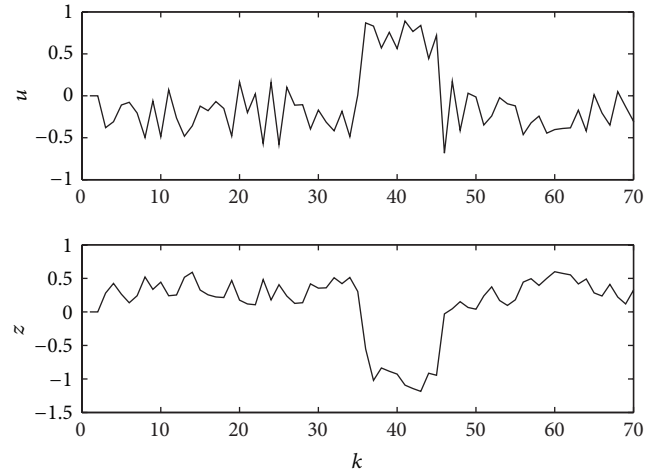


FIGURE 10: The time-history of the controlled output, z , and the control input, u , for $d_{\max} = 7$, $d_{\min} = 1$.

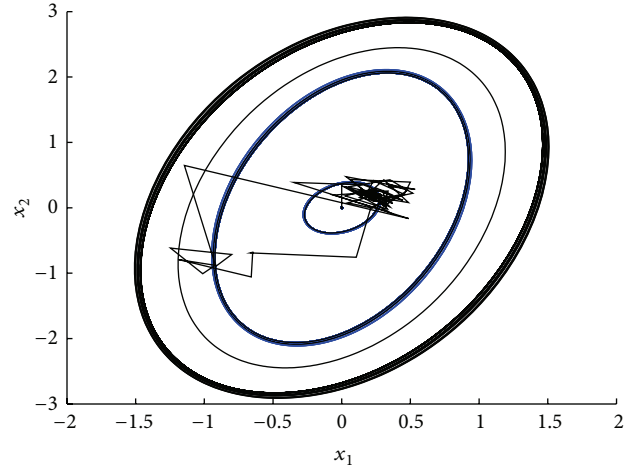


FIGURE 11: The ellipsoidal bounds on state trajectory of the uncertain system for $d_{\min} = d_{\max} = 2$.

considered. Utilizing a standard discrete-time Lyapunov-Krasovskii functional, some delay-dependent, linear matrix inequality (LMI) based conditions which need to be solved iteratively in each step of run-time were provided. The provided LMI based conditions guarantee the closed-loop asymptotic stability, maximum disturbance attenuation performance, and closed-loop dissipativity in consideration of the physical limitations of the actuator. Two numerical examples which consist of nominal and uncertain system models were considered to demonstrate the applicability of the proposed approach. Both the numerical results and simulation results validated that the proposed approach of this note can be efficiently used for the control of discrete-time, time-delayed systems having uncertainties and physical control limitations. For a further study, the conservatism of the approach can be alleviated by using complete *LK* functionals and/or some delay-decomposition methods. However, it is probable that both strategies would impose noticeable

amount of additional computational loads to the controller which will prohibit the use of the proposed method in real-time. Also these types of methods cannot be solved by using convex optimization methods and would require the use of some linearisation techniques such as cone-complementary methods.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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